1986 (H) P.E. Q10

(a) $x\frac{dy}{dx} = y(1-x) \Rightarrow \int \frac{dy}{y} = \int \frac{(1-x)}{x}dx + A$

$\Rightarrow \int \frac{dy}{y} = \int \left(\frac{1}{x} - 1\right)dx + A.$

$\Rightarrow \ln y = \ln x - x + A$

Now $y = 3$ when $x = 1$ $\Rightarrow \ln 3 = \ln 1 - 1 + A.$

$\Rightarrow A = \ln 3 + 1 \quad (\ln 1 = 0)$

$\Rightarrow \ln y = \ln x - x + \ln 3 + 1$

$\Rightarrow \ln y - \ln x - \ln 3 = 1 - x.$

$\Rightarrow \ln\left(\frac{y}{3x}\right) = 1 - x$

$\Rightarrow \frac{y}{3x} = e^{1-x} \Rightarrow \boxed{y = 3x e^{1-x}}$

(b)

P: $v = 15$, $t = 0$ ← $F = -mkv^3$ → Q: $v = 7.5\,ms^{-1}$, $t = t$

NII $\Rightarrow -mkv^3 = m\,accel$

$\Rightarrow \frac{dv}{dt} = -kv^3$ or $v\frac{dv}{dx} = -kv^3$.

Link v and time:

(1) $\Rightarrow \int_{15}^{7.5} \frac{dv}{v^3} = -\int_0^t k\,dt$

$\Rightarrow -\frac{1}{2v^2}\Big]_{15}^{7.5} = -kt$

$\Rightarrow \frac{1}{2v^2}\Big]_{15}^{7.5} = kt.$

$\Rightarrow \frac{1}{2(7.5)^2} - \frac{1}{2(15)^2} = kt$

$\Rightarrow \frac{1}{150} = kt$

ctd.

But we want to find distance gone in t secs when speed drops from 15 ms⁻¹ to 7.5 ms⁻¹. Let distance be X.

$v\frac{dv}{dx} = -kv^3$

$\Rightarrow \int_{15}^{7.5} \frac{dv}{v^2} = \int_0^X -k\,dx$

$\frac{-1}{v}\Big|_{15}^{7.5} = -kX.$

$\Rightarrow \frac{1}{v}\Big|_{15}^{7.5} = kX$

$\Rightarrow \frac{1}{7.5} - \frac{1}{15} = kX$

$\Rightarrow \underline{kX = \frac{1}{15}}$

$\therefore \frac{kX}{kt} = \frac{150}{15} \Rightarrow X = 10t$ qed.